Short Papers

Damping of the Resonant Modes of a Rectangular Metal Package

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Abstract —**When an electrical circuit is enclosed in a metal package large enough to support resonant modes within the frequencies of operation of the enclosed circuit, coupling between the circuit and these resonant cavity modes may disturb circuit operation. These resonant cavity modes can be effectively damped by placing a dielectric substrate coated with a resistive film in the cavity. In this paper a numerical algorithm is used to find the frequencies and quality factors of the lowest order resonant mode of a cavity damped in this manner.**

I. Introduction

Electrical circuits must often be enclosed in metal packages both to protect the circuit from material contamination and to provide electrical isolation. Many circuits, especially monolithic microwave integrated circuits (MMIC's), must be placed in metal packages which are large enough to support resonant modes at their frequencies of operation. (The frequencies of the resonant modes of a metal package decrease as the package dimensions increase, increasing the likelihood of interference with the enclosed circuit.) If these resonant modes have a very high quality factor Q, as is usually the case, even a very loose coupling between the circuit and these modes can disturb circuit operation.

This undesirable interaction between the circuit and the resonant cavity modes of the package can be reduced or eliminated by damping the resonant cavity modes. Conventional microwave absorbers composed of materials with bulk resistive properties may be placed in the package for this purpose, as has been done by Hallford and Bach [1]. Circuit reliability may be compromised, however, if microwave absorbers based on organic materials such as silicon rubber with a potential for outgassing are placed in the package with GaAs MMIC's. Furthermore, many microwave absorbers based on inorganic materials are difficult to machine to the small thicknesses required at microwave frequencies.

In this paper it will be shown that the resonant modes of a rectangular metal package may be damped by fixing a dielectric substrate coated with a thin resistive film to one of its walls, thus solving the reliability and machining problems associated with many conventional microwave absorbers. This is similar to the approach used in the Jaumann absorber [2], in which resistive films supported by low dielectric substrates are placed at roughly quarter-wavelength intervals from a ground plane to suppress electromagnetic reflections. The technique discussed here differs, however, in that the substrates in this case may have a high dielectric constant, may be much thinner, and are designed to suppress resonant modes rather than propagating waves.

The package which is investigated is shown in Fig. 1. The package is assumed to have perfectly conducting metal walls. A dielectric substrate with relative dielectric constant ϵ_r and thickness t is fixed to the upper wall of the package cavity. A thin resistive film coats the air–dielectric interface at $z = h - t$.

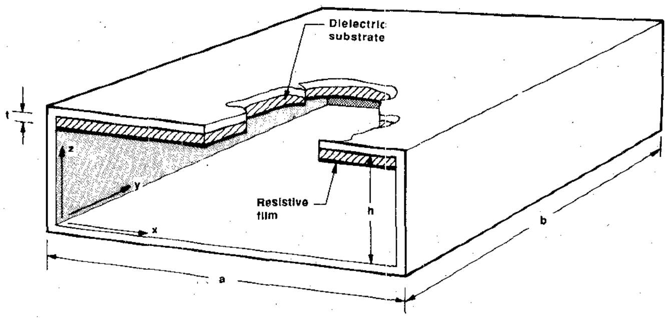

Fig. 1. Dimensions of the rectangular metal package. The substrate has a dielectric constant ϵ_r and a thickness t. The substrate is coated with a thin film of resistivity R_m at $z = h - t$.

In the following it is shown how the fields in the cavity of the package may be expressed as a superposition of simple modes. It is then shown how consideration of the boundary conditions at the air–dielectric interface supporting the resistive film leads to a numerical algorithm for determining the resonant frequency and the quality factor Q of the resonant cavity modes. Finally, for a number of geometries, graphs are given allowing the determination of the film resistivity which optimally damps (i.e., reduces the dominant resonant mode's unloaded quality factor to its minimum value) the lowest order cavity mode of packages with heights much less than their widths or lengths.

II. The Boundary Value Problem

In the absence of the resistive film, the resonant cavity modes of the package are the TM_{nml} and TE_{nml} modes discussed by Harrington [3] with respect the z coordinate defined in Fig. 1, where n, m, and l are the mode numbers in the x, y, and z directions, respectively. (It is not possible to find TM or TE modes with respect to x or y, as discussed by Harrington [3].) We will restrict our attention to these modes, and will only discuss the TM_{110} mode in detail. (The TM_{110} mode is the first mode to be excited in a package with a height much less than its length or width and, therefore, is in practice the most often encountered resonant mode.)

For substrate dielectric constants not equal to unity, the TM_{110} mode has electric field components tangential to the air–dielectric interface, and can be damped by a resistive film at that interface. It is desirable to choose a film resistivity which will damp this mode as fully as possible (i.e., reduce its unloaded quality factor Q to the minimum possible value). It is not possible to use perturbation techniques to estimate the film resistivity required to optimally damp this resonant mode because the resistive film alters the fields of the mode significantly when it is maximally damped. It is possible, however, to solve for the resonant modes of the package directly, as will be outlined below.

The TM_{nml} and TE_{nml} modes of the package cavity may be treated by writing the fields in the dielectric and in the air as a superposition of TM_{nm} and TE_{nm} modes, respectively. (The TM_{nm} and TE_{nm} modes are not coupled by the boundary conditions at the air–dielectric interface, even in the presence of a

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resistive film.) The resonant frequencies and relative amplitudes of the TM_{nm} or the TE_{nm} modes in each region may be determined by matching the boundary conditions at the air–dielectric interface.

If the time variation $e^{j\omega t}$ is suppressed, the TM_{nm} modes with respect to z below the air–dielectric interface ($0 < z < h - t$) are [4]

$$E_z = A_{nm} \sin(n\pi x/a) \sin(m\pi y/b) \cos(\beta_{nm} z) \tag{1}$$

$$E_x = -A_{nm}(\beta_{nm}(n\pi/a)/k_{cnm}^2) \cos(n\pi x/a) \cdot \sin(m\pi y/b) \sin(\beta_{nm} z) \tag{2}$$

$$E_y = -A_{nm}(\beta_{nm}(m\pi/b)/k_{cnm}^2) \sin(n\pi x/a) \cdot \cos(m\pi y/b) \sin(\beta_{nm} z) \tag{3}$$

$$H_z = 0 \tag{4}$$

$$H_x = A_{nm}(jk_0/Z_0 k_{cnm}^2)(m\pi/b) \sin(n\pi x/a) \cdot \cos(m\pi y/b) \cos(\beta_{nm} z) \tag{5}$$

$$H_y = -A_{nm}(jk_0/Z_0 k_{cnm}^2)(n\pi/a) \cos(n\pi x/a) \cdot \sin(m\pi y/b) \cos(\beta_{nm} z) \tag{6}$$

and the TE_{nm} modes with respect to z are

$$H_z = C_{nm} \cos(n\pi x/a) \cos(m\pi y/b) \sin(\beta_{nm} z) \tag{7}$$

$$H_x = -C_{nm}(\beta_{nm}(n\pi/a)/k_{cnm}^2) \sin(n\pi x/a) \cdot \cos(m\pi y/b) \cos(\beta_{nmz}) \tag{8}$$

$$H_y = -C_{nm}(\beta_{nm}(m\pi/b)/k_{cnm}^2) \cos(n\pi x/a) \cdot \sin(m\pi y/b) \cos(\beta_{nm} z) \tag{9}$$

$$E_z = 0 \tag{10}$$

$$E_x = C_{nm}(jk_0 Z_0(m\pi/b)/k_{cnm}^2) \cos(n\pi x/a) \cdot \sin(m\pi y/b) \sin(\beta_{nm} z) \tag{11}$$

$$E_y = -C_{nm}(jk_0 Z_0(n\pi/a)/k_{cnm}^2) \sin(n\pi x/a) \cdot \cos(m\pi y/b) \sin(\beta_{nm} z) \tag{12}$$

where

$$\beta_{nm}^2 = k_0^2 - k_{cnm}^2 \tag{13}$$

$$k_{cnm}^2 = (n\pi/a)^2 + (m\pi/b)^2 \tag{14}$$

$$k_0 = \omega/c \tag{15}$$

and Z_0 is the impedance of free space, c is the speed of light in free space, ω is the resonant frequency of the mode, and A_{nm} and C_{nm} are the amplitudes of the TM_{nm} and the TE_{nm} modes in the cavity region, respectively. Since the cavity is lossy, ω may be complex.

The electromagnetic fields in the dielectric region ($h - t < z < h$) are described as a superposition of the same TE_{nm} and TM_{nm} modes, except that β_{nm} is replaced by β'_{nm}, A_{nm} is replaced by B_{nm}, C_{nm} is replaced by D_{nm}, (5) and (6) are multiplied by ϵ_r, and z is replaced by $z - h$, where

$$\beta'^2_{nm} = \epsilon_r k_0^2 - k_{cnm}^2 \tag{16}$$

and where B_{nm} and D_{nm} are the amplitudes of the TM_{nm} and the TE_{nm} modes in the dielectric, respectively.

At the dielectric interface, the tangential electric field must be continuous. This implies that

$$E_t^- = E_t^+ \tag{17}$$

where E_t^+ and E_t^- are the electric fields tangential to and just above and below the dielectric interface, respectively. The tangential electric field at the interface can be related to the surface current at the interface via Ohm's law. The discontinuity in the magnetic field across the interface can also be related to the surface current, giving a second boundary condition

$$E_t = R_m \hat{z} \times (H^+ - H^-) \tag{18}$$

where R_m is the surface resistivity in Ohms per square of the film, z is the unit vector in the z direction, $\times$ denotes the vector cross product, H^+ and H^- are the magnetic fields just above and below the air–dielectric interface, respectively, and $E_t = E_t^+ = E_t^-$ (see (17)) is the tangential electric field at the interface.

To find the solution for a TM_{nml} or a TE_{nml} mode, the expressions for the fields of the TM_{nm} or the TE_{nm} modes, respectively, at the air–dielectric interface given in (1)–(16) are inserted into (17) and (18). It is easy to show that when the x component of (17) and (18) are satisfied, the boundary conditions on the y and z components of the magnetic and electric fields are also satisfied. Thus only one tangential component of (17) and (18) need be considered. If common factors are eliminated, the result may be expressed in matrix form as

$$[M_{ij}] \cdot \begin{bmatrix} A_{nm} \\ B_{nm} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{19}$$

for the TM modes and

$$[K_{ij}] \cdot \begin{bmatrix} C_{nm} \\ D_{nm} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{20}$$

for the TE modes where the x component of (17) results in

$$M_{11} = -\beta_{nm}(n\pi/a) \sin(\beta_{nm}(h-t))$$
$$M_{12} = -\beta'_{nm}(n\pi/a) \sin(\beta'_{nm} t)$$
$$K_{11} = jk_0 Z_0(m\pi/b) \sin(\beta_{nm}(h-t))$$
$$K_{12} = jk_0 Z_0(m\pi/b) \sin(\beta'_{nm} t)$$

and where the x component of (18) results in

$$M_{21} = M_{11} + (jk_0/Z_0) R_m(n\pi/a) \cos(\beta_{nm}(h-t))$$
$$M_{22} = -\epsilon_r(jk_0/Z_0) R_m(n\pi/a) \cos(\beta'_{nm} t)$$
$$K_{21} = K_{11} + \beta_{nm} R_m(m\pi/b) \cos(\beta_{nm}(h-t))$$
$$K_{22} = -\beta'_{nm} R_m(m\pi/b) \cos(\beta'_{nm} t).$$

Equations (19) and (20) are complex transcendental equations. Equation (19) may be written as

$$\epsilon_r \beta_{nm} \tan(\beta_{nm}(h-t)) \frac{1}{1-E} = -\beta'_{nm} \tan(\beta'_{nm} t) \tag{21}$$

where E is given by $E = \beta_{nm} \tan(\beta_{nm}(h-t))/(R_m jk_0/Z_0)$ and (20) may be written as

$$\beta'_{nm} \tan(\beta_{nm}(h-t)) \frac{1}{1+F} = -\beta_{nm} \tan(\beta'_{nm} t) \tag{22}$$

where F is given by $F = (jk_0 Z_0) \tan(\beta_{nm}(h-t))/(R_m \beta_{nm})$. Equations (21) and (22) reduce to the usual characteristic equations (see [3, eqs. (4-45), (4-47)]) for the TM_{nml} and the TE_{nml} modes, respectively, when R_m is large.

In general, neither equation has a nonzero solution. If either M or K are singular, a nonzero solution exists, the determinant of the singular matrix is zero, and either (21) or (22) is satisfied. In this case, the nontrivial solution corresponds to a TM_{nml} or a TE_{nml} resonant cavity mode. The determinants of M and K are complex functions of ω, and hence their zeroes are not easily found analytically. These determinants are easily calculated on a computer, however, as a function of ω. The zeroes of the determi-

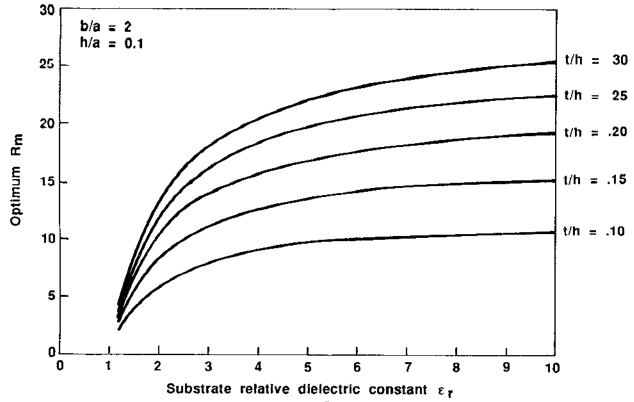

Fig. 2. The film resistivity which damps optimally the TM_{110} mode is plotted as a function of the substrate dielectric constant ϵ_r for several values of normalized substrate thickness t/h.

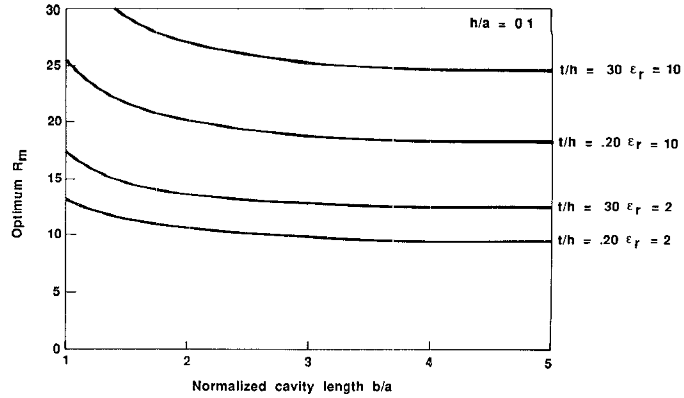

Fig. 4. The film resistivity which damps optimally the TM_{110} mode is plotted as a function of the normalized cavity length b/a for several values of normalized substrate thickness t/h and dielectric constant ϵ_r.

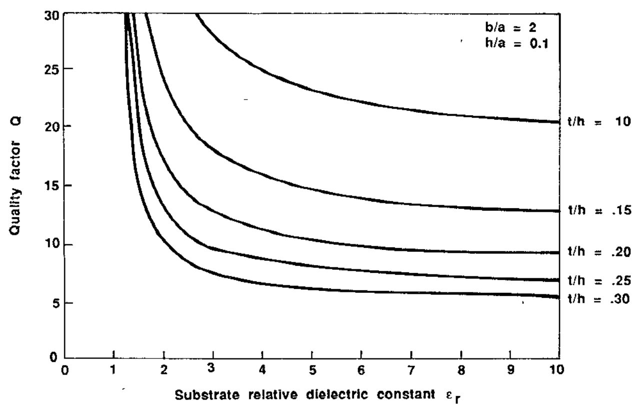

Fig. 3. The Q of the optimally damped TM_{110} mode is plotted as a function of the substrate dielectric constant ϵ_r for several values of normalized substrate thickness t/h.

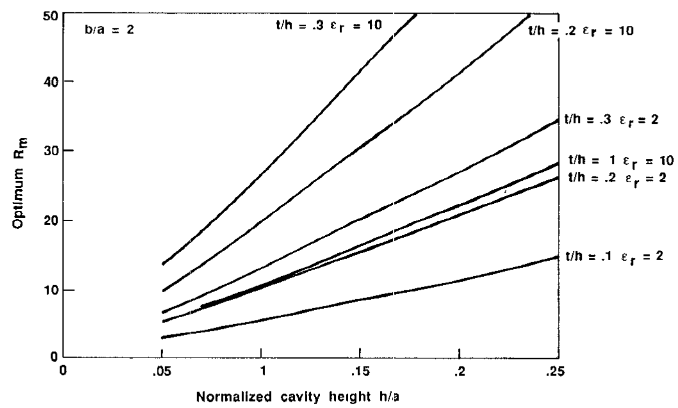

Fig. 5. The film resistivity which damps optimally the TM_{110} mode is plotted as a function of the normalized cavity height h/a for several values of normalized substrate thickness t/h and dielectric constant ϵ_r.

nants may then be found by an algorithm employing a gradient search technique using the resonant frequency of the TM_{nml} mode at infinite resistivity as a starting value. If the real and imaginary parts of the ω for which the determinate of M or K is zero are given by ω_r and ω_i, respectively, then the frequency of the resonant mode is just ω_r and the rate at which the mode is damped is determined by ω_i. Neglecting the loading introduced by the coupling of the circuit to the mode, the quality factor Q of the resonant mode (often referred to as the unloaded quality factor) is adequately approximated as [5]

$$Q \approx \omega_r/(2\omega_i) \tag{23}$$

for reasonably large values of Q. This equation is used for the present calculations.

III. Numerical Results

The film resistivity which damps the TM_{110} mode optimally is of special interest since this is the mode with the lowest resonant frequency when the cavity height is much less than its width or length. In Fig. 2 the film resistivity which damps the TM_{110} mode optimally is plotted for a cavity with dimensions $b/a = 2$ and $h/a = 0.1$ as a function of ϵ_r for several substrate thicknesses. The Q of the optimally damped mode is plotted in Fig. 3. It was found that the resonant frequencies of the mode changed only minimally when damped optimally for the cases considered here in which the substrate thickness was small compared to the package height, and thus were not plotted. From Fig. 3 it is apparent that substrates with high dielectric constants are more effective in damping the mode. This is because the tangential electric field of the mode, which is responsible for the surface currents in the resistive film, decreases as the dielectric constant of the substrate ϵ_r approaches unity, and vanishes in the limit of $\epsilon_r = 1$. In Fig. 4 this optimal film resistivity is shown as a function of the normalized cavity length b/a for two values of substrate thickness and dielectric constant. In Fig. 5 the optimal resistivity is shown as a function of the normalized cavity height h/a for several values of substrate thickness and dielectric constant. The Q of the optimally damped mode shows very little dependence on the normalized cavity length b/a or the normalized cavity height h/a, and can be estimated to within 20 percent from Fig. 3 over the ranges plotted.

IV. Conclusion

It has been shown that the lowest order TM_{110} resonant mode of a metal package can be suppressed by fixing a substrate coated with a thin resistive film to the upper wall of the package cavity. Design information was presented to easily allow unwanted modes to be effectively damped with this technique.

It was found that it is possible to obtain resonant mode quality factors of 20 or less with film resistivities between 10 and 30 Ω/sq on high dielectric constant substrates. Thin films of this resistivity can be formed on alumina, which has a high dielectric constant, by evaporating approximately 100–1000 Å of chrome or titanium on its surface. The backside of the alumina substrate may then be metalized and soldered to the lid of a package containing a large MMIC, using the same proven technology used to affix the circuit to the bottom of the package, effectively damping the lowest order cavity mode supported by the package. This is an inexpensive alternative to conventional microwave absorbers, and has the advantage of posing no circuit reliability problems.

The results presented were limited to the TM_{110} mode. The analysis may be easily applied to higher order cavity modes if the circuit couples to those modes.

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Finite Element Formulation for Guided-Wave Problems Using Transverse Electric Field Component

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Abstract **—A finite-element formulation for electromagnetic waveguide problems is described using the transverse electric field component. In this approach, the divergence relation $\nabla \cdot \boldsymbol{D} = 0$ is satisfied and spurious solutions can be eliminated in the entire region of a propagation diagram. The validity of the formulation is examined via applications to a few canonical guided-wave problems.**

I. INTRODUCTION

The most serious difficulty in applying the finite element method to waveguide problems has been the appearance of so-called spurious, nonphysical solutions. To overcome this difficulty, various approaches have recently been developed; these are reviewed in [1]. More recently, a new finite element formulation for the analysis of dielectric waveguide modes has been developed by the authors in terms of the transverse magnetic field component ($\boldsymbol{H}_t$) [2]. The key point of this method, which is distinctly different from other transverse field methods [3]–[7], is that it transforms the finite element equation in terms of a full vector $\boldsymbol{H}$ field [8], [9] into one in terms of only the transverse magnetic field component, using the condition $\nabla \cdot \boldsymbol{H} = 0$. In this approach, the spurious solutions can be completely eliminated in the entire region of a propagation diagram, and the final matrix dimension is reduced to two thirds that of the conventional three-component approach using the penalty function method [10]–[14]. However, in the finite element analysis based on this approach, the magnetic field components are first obtained as an eigenvector, and the electric field components are later derived from them via Maxwell's equations. This additional operation based on spatial differentiations of the original data may cause an unnatural field profile when one uses lower order Lagrange elements.

In this paper, as an electric field version of the method described in [2], a finite element method for electromagnetic waveguide problems is formulated using the transverse electric field component ($\boldsymbol{E}_t$). In this approach, the spurious solutions can be eliminated in the entire region of a propagation diagram, and the dimension of the final matrix equation is reduced to two thirds that of the full vector $\boldsymbol{E}$ field approach based on a penalty function [15], [16]. The validity of the present method is confirmed via applications to a few representative waveguiding problems.

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II. FORMULATION

We consider a waveguide with a tensor permeability and a scalar permittivity. With a time dependence of the form $\exp(j\omega t)$ being implied, from Maxwell's equations the following wave equation is derived:

$$\nabla \times \left([\mu]^{-1} \nabla \times \boldsymbol{E}\right) - k_0^2 \epsilon \boldsymbol{E} = 0 \tag{1}$$

where ω is the angular frequency, k_0 is the free-space wavenumber, $[\mu]$ is the relative permeability tensor, and ϵ is the relative permittivity, which is assumed to be constant in each material.

The divergence relation for source-free media, $\nabla \cdot \boldsymbol{D} = 0$, can be written

$$\epsilon E_z = (j\beta)^{-1}\left(\epsilon \partial E_x/\partial x + \epsilon \partial E_y/\partial y\right) \tag{2}$$

where β is the phase constant in the propagation direction (z direction).

Application of the standard finite element technique [2] to (1) and (2) gives the following matrix equations:

$$[S]\{E\} - (k_0/\beta)^2[T]\{E\} = \{0\} \tag{3}$$

$$[D_z]\{E_z\} = [D_t]\{E_t\} \tag{4}$$

where

$$[S] = \sum_e \iint_e [B]^*[\mu]_e^{-1}[B]^T \, d\bar{x}\, d\bar{y} \tag{5}$$

$$[T] = \sum_e \iint_e \epsilon_e [N]^*[N]^T \, d\bar{x}\, d\bar{y} \tag{6}$$

$$[D_z] = \sum_e \iint_e \epsilon_e \{N\}\{N\}^T \, d\bar{x}\, d\bar{y} \tag{7}$$

$$[D_t] = -\sum_e \iint_e \left[\epsilon_e \{N\}\{N\}_x^T \quad \epsilon_e \{N\}\{N\}_y^T\right] d\bar{x}\, d\bar{y} \tag{8}$$

$$\{E_t\} = \begin{bmatrix} \{E_x\} \\ \{E_y\} \end{bmatrix}. \tag{9}$$

Here, $\{N\}$ is the shape function vector; $\{0\}$ is a null vector; T, $\{\cdot\}$, and $\{\cdot\}^T$ denote a transpose, a column vector, and a row vector, respectively; the components of vectors $\{E_x\}$, $\{E_y\}$, and $\{E_z\}$ are the values of E_x, E_y, and E_z at nodal points in the cross section, respectively; * denotes complex conjugate; $\bar{x} = \beta x$ and $\bar{y} = \beta y$; and $[N]$ and $[B]$ are given in [14].

Using (4), we can express the nodal electric field vector $\{E\}$ in terms of $\{E_t\}$:

$$\{E\} = [D]\{E_t\} \tag{10}$$

where

$$[D] = \begin{bmatrix} [U] \\ [D_z]^{-1}[D_t] \end{bmatrix}. \tag{11}$$

Here $[U]$ is a unit matrix.

Substituting (10) into (3) and multiplying (3) by $[D]^T$ from the left, we obtain the following final matrix equation with respect to the transverse electric field component $\{E_t\}$:

$$[\tilde{S}_{tt}]\{E_t\} - (k_0/\beta)^2[\tilde{T}_{tt}]\{E_t\} = \{0\} \tag{12}$$